Decoding emotions in relation to perceived difficulty: How primary students engage with mathematical tasks

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This study explores emotional factors influencing primary school students' perceived difficulty when solving mathematical tasks. The research involved 49 students aged 7 to 9, with 20 of them being interviewed. The aim was to identify emotions related to perceived task difficulty through qualitative analysis of semi-structured interviews. Students engaged in collaborative, geometry-focused activities using Lénárt spheres. The results indicate that while emotional factors, both positive and negative, emerged, students often struggled to articulate these emotions. The findings suggest that emotions are meaningful and independent contributors to perceived difficulty. Further research is required to deepen understanding of the role emotions play in students' learning experiences.

Keywords: Mathematics education, qualitative study, perceived difficulty, emotions, primary school.

Introduction and theoretical framework

Research in mathematics education has long dealt with difficulties in mathematics. Focusing on students' difficulties related to solving mathematical tasks, factors such as the mathematical content and affective factors have an influence on them (see, for example, Zan, et al., 2006). On the contrary, the issue of perceived difficulty has not been analysed with the same attention. The *difficulty* and the *perceived difficulty* of a task are deeply interconnected; however, they are different. In fact, the difficulty of a task is usually objective and evaluated in retrospect based on the results obtained from the sample to which the task was proposed (Mehrens & Lehmann, 1991), while perceived difficulty is related to individual perception, and it is more personal. In mathematics education, there is currently no agreed-upon definition of the perceived difficulty of a task, although this aspect has been analysed in different fields, including cognitive psychology (see, for example, Efklides & Touroutoglou, 2010).

Concerning students, there are very few studies analysing their perception of difficulty and the factors influencing it and, in those studies, difficulty and perceived difficulty are not explicitly distinguished and the focus is on the cognitive side of the process, meaning that the factors described are merely linked to the objective characteristics of the task (Lee & Heyworth, 2000). Recently, some qualitative research highlighted the factors that contribute to the perceived difficulty according to students with respect to mathematical tasks (Saccoletto & Spagnolo, 2022; Spagnolo & Saccoletto, 2023). Those factors were organised into five non mutually exclusive macro-categories (Resolution Strategies, Capabilities and Experience, Emotions, Task Formulation, Self-considerations), further detailed in more specific subcategories. The macro-categories come from textual analysis of secondary students' open-ended answers, when asked to solve mathematical tasks, evaluate their difficulty and explain their reasons. The analysis was performed according to Constructivist Grounded Theory (Mills et al., 2006), therefore the macro-categories of factors influencing perceived difficulty have been

determined inductively from data. The first macro-category is “Resolution strategies” and it refers to the strategies necessary to solve the task or the fact that elements were needed to reach a solution. The second macro-category is “Capabilities and experience”, and it includes students’ self-perceived competence, skills as well as eventual prior experiences in solving similar tasks, and therefore their familiarity with them. Within this macro-category, we can find also students’ reflections on their doubts and challenges faced during problem-solving and the time spent doing it. The third macro-category, named “Emotions”, regards the emotional aspects. The fourth macro-category, “Task formulation”, refers to the formulation of the task itself, both in its textual and non-verbal elements. Finally, the fifth macro-category is “Self-considerations”, and it includes references to students’ personal reflections related to their own success and effort in mathematics. Qualitative research involving secondary school students, carried out using the macro-categories, showed that the “Emotions” one results systematically underrepresented or not represented at all (Spagnolo & Saccoletto, 2023; Nicchiotti & Spagnolo, 2024). In fact, secondary school students seem not to give importance to subjective aspects when dealing with mathematics and tend to mention the emotional side only when directly asked about it (Nicchiotti & Spagnolo, 2024).

Research in mathematics education has shown an increasing interest in affective aspects concerning mathematics from the beginning of the 2000s while, until then, it had focused mostly on cognitive factors, with rare exceptions such as mathematics anxiety (Schukajlow et al., 2023). McLeod (1992) conceptualizes emotions in mathematics learning considering them as part of an affective range together with belief and attitudes, with emotions being the most intense but the least stable among the three. Furthermore, Ekman’s theory of basic emotions (1992) has been used in research about problem solving in mathematics education (Hannula, 2015), but it emerged that basic emotions would be rare in learning context. Therefore, there is the need to investigate emotions emerging strictly in the educational context (Lehman et al., 2008). We referred to achievement emotions (Pekrun, 2006, 2011), which are defined as emotions linked to activities or outcomes, evaluated according to achievement-related quality standards. In particular, we adopted the novel three-dimensional taxonomy of achievement emotions (Pekrun et al., 2023), which includes the following 12 emotions: enjoyment, hope, pride, relaxation, assurance, relief, anger, anxiety, shame, boredom, hopelessness, disappointment. According to Pekrun and colleagues (2023), individual perception, including perceived difficulty of a task, can influence emotions. However, emotion can mutually impact the environment and the perceptions associated with it, even though this side of the reciprocal causation still needs to be investigated (Pekrun et al., 2023).

Research aim

Under these premises, it appears that students’ perceived difficulty needs to be investigated further, focusing on the possible emotional factors influencing it. Antognazza and colleagues (2015) show that primary school students, when asked straightforward questions about the emotions felt before and during problem solving, mention different emotions, both positive and negative; these emotions are related to their assessment of the difficulty of the problem (Antognazza et al., 2015). We are interested in exploring whether emotions would still emerge with different and more open prompts, not explicitly mentioning them, and check if students’ emotions would allow us to characterize more precisely the concept of perceived difficulty.

In this first explorative study, we focused on primary school students, as research still needs to explore the emotions experienced by children in that age group (Lichtenfeld et al., 2012). Our research question can be expressed as follows: which types of emotion related to perceived difficulty emerge, if any, from primary school students when dealing with mathematical tasks?

Methods

We carried out a qualitative exploratory study, involving 49 primary school students aged 7 to 9, who were attending an optional after-school course. The students were 21 girls and 28 boys, and they were from different classes and had been grouped into two new classes for the course (hence, each of them knew only some of the other children). They participated in five sessions, lasting two hours each, held over the course of a week. The activities were guided by two researchers in mathematics education; the mathematics class teachers were present only to supervise the students and they interacted sporadically to voluntarily participate in some of the activities with the students. The course consisted of activities with active involvement of the learner, in the sense proposed by Emma Castelnuovo (see, e.g., Castelnuovo, 1963).

The course was titled "The geometry of soap bubbles" and was focused on activities regarding comparative geometry, namely the comparison between Euclidean and spherical geometry in our case. During the sessions, the students worked in small groups collaboratively using Lénárt spheres to reflect on geometrical concepts and their characterisation on the plane and on the sphere; they investigated and built geometrical objects and definitions. The tasks proposed were selected among the ones experimented with children from different school grades and pre-service teachers using the Lénárt spheres, already present in the literature (Gambini, 2021; Gambini & Lénárt, 2021). Some of the tasks required children to discover the counterpart of segments, lines and triangles (which they knew on the plane) on the sphere. The tasks were introduced in a discursive and playful manner: we started recollecting the knowledge the students already had about plane geometry; then students were asked what they thought would happen if they tried to construct the same geometric entities on the sphere. They found out that lines on the sphere are maximum circumferences, and therefore on the sphere it is indeed possible to build three perpendicular lines, and that on the sphere the sum of the internal angles of a triangle could be greater than 180°. Much time was dedicated to their discussion and explanation of the construction and reasoning of each group.

At the end of the last session, we randomly selected 20 students and interviewed them in 7 small groups, keeping together the ones that had worked together during the sessions. Each group of students was interviewed separately; we let them express their opinions freely and sometimes asked clarifying questions. We asked them how they perceived the tasks, if they were easy or difficult and why, then we asked what they would change, being the teacher, to make the tasks easier. We decided to evaluate perceived difficulty after solving the task as we were interested also in the elements related to strategies implemented to solve it. We recorded the audio of the interviews and later analysed the transcript qualitatively, focusing on the answers to the first questions. The analysis was carried out using the software MAXQDA Analytics Pro 24 (v. 0.4). Specifically, we imported the interview transcripts into the software, where they appeared in the "Documents" section. Subsequently, we created 17 distinct codes within the "Code" section, each corresponding to a specific subcategory

within the five macro-categories of factors influencing perceived difficulty (Spagnolo & Saccoletto, 2023). Each macro-category's code was assigned a unique and casual colour and each code was labeled according to its respective subcategory. Following this preparatory phase, we proceeded with the analysis of each interview by reading and identifying words or sentences that were relevant for our research, and assigning each excerpt an appropriate code. Upon completion of this step, the "Code" section displayed numerical values next to each code, indicating the frequency of the code application throughout the dataset. Finally, we analysed in depth any reference to the macro-category "Emotions", using achievement emotions (Pekrun, 2023) as a reference, highlighting each specific emotion expressed by students. The coding phase was carried out independently by two of the researchers, who then compared their results with each other and with the third researcher to reach a consensual coding.

Results and discussion

We started analysing children' perceived difficulty of the tasks proposed. The answers varied from "some [tasks were] easy and some difficult", "medium difficulty" and "differently easy". None of the children stated that they were all easy or difficult and most of them then provided punctual examples of tasks solved during the sessions that they found easy or difficult. We noticed that there was no accordance between the children in defining the same activity easy or difficult, even with children from the same small group. Each student had their own perceived difficulty regarding each one of the tasks, and their perceived difficulty was influenced, but not determined, in absolute terms by the group they worked in.

Regarding the factors influencing perceived difficulty, we could find reference to each of the five macro-categories. In Table 1 we reported a summary with the number of references classified in each of them and the percentage. Being the macro-categories non mutually exclusive, the same part of the transcript might be categorised in more than one of them and therefore coded with different codes. It occurred with high frequency to find short statements in which elements pertaining to different macro-categories were mentioned. A representative example is the phrase "[Drawing three parallel lines on the sphere was difficult] because I had drawn a circle, another one on it and one more had to be done crossway, and I had done it kind of sideways", which belongs to the macro-category "Capabilities and experience" because of the reference to the obstacle encountered but also to "Resolution strategy" because the child explains the procedure needed to solve the problem.

Table 1: Distribution of the references among the macro-categories

Macro-categories	Number of references	Percentage
Resolution strategies	11	22,45%
Capabilities and experience	17	34,69%
Emotions	6	12,24%
Task formulation	5	10,20%

Self-considerations	10	20,41%

As shown in Table 1, we identified 6 references to the macro-category "Emotions". The emotional factors were mentioned by 4 out of 7 groups interviewed: only once, the fact that a student mentioned emotions during the interview sparked a discussion in the group about the theme, resulting in more children considering them. Moreover, the emotional factors do not seem to be systematically related to other specific macro-categories, meaning that the children who mention emotions consider also other factors when explaining their perceived difficulty, but those factors vary depending on the child.

Moving on to describe the emotions emerged, we identified both positive and negative emotions, which coexist even within the same group of children. In Table 2 we show the parts of the transcript coded with the code "Emotion", associated with the specific achievement emotion that we assigned during the analysis.

Table 2: Interview excerpts referring to emotions, associated with the achievement emotion involved

Excerpt	**Achievement emotion**
[The task regarding the spherical triangle was difficult] because Child A did everything, because I had not understood [how to do it].	Shame
[The activities] were fun.	Enjoyment
Because I like geometry, well, the subjects I like are also easier in my opinion.	Enjoyment
[Working] alone is easier but together is more fun.	Enjoyment
[Working] alone [is easier] because alone you are more satisfied	Pride
[It was hard because] the first time I said oh my dear! spherical geometry what is that?	*Hopelessness*

Not all the achievement emotions have been identified in the interviews; however, the model seems suitable for our analysis as the emotions expressed by children were classifiable as such. The only exception is the last excerpt. It was part of a student's longer and broad discourse regarding also their previous experiences with mathematics and other school subjects, in which they said that when you see something for the first time you might think it is very hard but then you look back at it, you study and understand it perfectly. In this case, the emotions are conveyed through interjections and exclamations rather than a specific word or expression. We could identify this emotion as *hopelessness*, but it does not seem to fit precisely, as the phrase in its context seems to communicate something more similar to surprise, which is not part of the achievement emotions.

Another aspect to consider is the influence of the type of task on the emotions experienced by children. Firstly, not all children consider the type of task when motivating their perceived difficulty.

They discuss the task, but do not really consider explicitly their general characteristics, such as the fact that it is very distant from their everyday schoolwork, even though they are absolutely clear about that when asked if they have already done something similar. Secondly, the students that elaborate on the type of task and link it to emotional aspects, do it in different ways. For instance, two children say that working alone is easier: one of them links the autonomous work with a positive emotion (pride), while the other links the group work with a different positive emotion (enjoyment), even if they find it more difficult. Hence, the type of activities (in the sense of Emma Castelnuovo) seems to encourage the development of positive emotions, but it is not the only condition to do so.

Analysing the coded excerpts as a whole, we could finally make a general observation: children seem not to be able to describe their emotions, nor to give them a name. To communicate them, they rely on stories, mimical expressions and tones, more than words. This might be for two main reasons: the first possibility is that their young age makes it difficult to them to understand their emotional states; the second one is that their emotions are clear to them, but they lack the verbal skills and terminology to communicate them. Obviously, since the sample is very small, we cannot generalise our findings; however, they might be hypothesis to be taken into account and investigated with further research.

Concluding remarks

In this paper, we presented results of a first qualitative study aimed to explore if and how emotions are mentioned in relation to perceived difficulty by primary school children. To do so we focused on tasks which are far from the usual schoolwork, and we investigated children's opinions through semi-structured interviews. We could conclude that emotions are expressed among the factors influencing primary school children's perceived difficulty. In addition to that, both positive and negative emotions are considered (in accordance with Antognazza et al., 2015), and they do not seem to be subordinated to other factors. This means that emotions were present in children's responses independently of the presence of other factors and not only associated with some of them, suggesting that they are one of the independent macro-categories of factors influencing perceived difficulty also for primary school students. To generalize our findings, we need to be cautious due to the small sample size, and further research is required to validate and strengthen the results.

The need for the study originated from the underrepresentation of the macro-category "Emotions" in other qualitative studies (Spagnolo & Saccoletto, 2023; Nicchiotti & Spagnolo, 2024), while the findings of the present research are contradicting that. This might depend on several elements: the activity proposed, the age of the participants and the method of data collection. In fact, in the other studies, the tasks were not collaborative, with active involvement of the learner and distant from the usual ones done in class. Secondly, we involved primary school students, while in the other studies only high school students participated. Finally, in the other studies perceived difficulty had always been analysed through questionnaires of different types, and not always associated to interviews. We do not want to reduce the complexity of such an image isolating one key element to unlock the expression of emotions, nor we think it would be useful or possible. Anyway, the elements discussed have to be held present in successive investigations.

Finally, we see that the research on emotions as factors influencing perceived difficulty might be developed in many directions. Looking at the whole picture, we have primary school children which

express emotions but often struggling and they have difficulties in describing and naming their emotions. On the other hand, previous research shows that secondary school students often neglect emotions, not even mentioning them at all (Nicchiotti & Spagnolo, 2024). Therefore, there is the need to close the gap studying what happens in between the two moments. It is important to understand whether students simply lose or not develop the ability to describe their emotions as they grow older, or if they do not express emotions because they have still difficulties doing so as in primary school, but growing up they come to disregard emotions, because anything they have difficulties expressing basically does not exist.

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